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Curvature Corrections to Reynolds Stress Model for Computation of Turbulent Recirculating Flows

Sang Keun Dong*
Korea Institute of Energy Research, Taejon, Republic of Korea
and
Myung Kyoon Chung†
Korea Advanced Institute of Science and Technology, Taejon, Republic of Korea

Introduction

THE effect of streamline curvature on third-order velocity correlation has been experimentally investigated by Chung et al.[1] They found that the third-order correlation $\overline{u_i u_i u_j}$ in a curved-streamline field can be effectively represented by the simple gradient transport model with a model coefficient as a function of the ratio between the velocity time scale $\tau_v = k/\epsilon$ and a curvature time scale $\tau_c = \epsilon/(N^2 k)$, where $N^2 = 2(U/R)/(U/R + \partial U/\partial n)$ is the frequency squared of small oscillations of a fluid element displaced radially in a flow with a radius of curvature R. Park and Chung[2] adopted such a curvature correction to the third-order terms $\overline{kv}$ and $\overline{\epsilon v}$ and to the isotropic decay constant $C_{\epsilon 2}$ in the standard k-ϵ equations. Their curvature-dependent k-ϵ model was found satisfactory for predictions of various kinds of separated recirculating turbulent flows. More recently, Park and Chung[3] extended the curvature corrections to the Reynolds stress model for the computation of a turbulent flow over a mildly curved axisymmetric body. During the review process of the paper, one of the reviewers raised a serious question about the necessity of curvature correction to the Reynolds stress model (RSM). In fact, the RSM has been frequently applied to recirculating flows of high streamline curvature without any curvature correction. But it is noted that most of the numerical solutions by the conventional RSM show poor predictions with severe zonal dependence.[4,5] Since the streamlines are mildly curved in the test flow of Park and Chung,[3] the computational improvement by the curvature corrections is not sufficiently demonstrated.

The purpose of the present study is to examine more clearly the necessity of the curvature corrections to the RSM. Test flows selected here for comparisons are the backward-facing step flows of Pronchick[6] and Driver and Seegmiller.[7]

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*Research Engineer, P.O. Box 5, Daedeok Science Town.

†Professor, Department of Mechanical Engineering, 373-1 Kusong-dong, Yusung-Ku.

Turbulence Models

The turbulent transport $(\overline{u_i u_j u_k})$ in the Reynolds stress equation is approximated by the gradient transport model of Hanjalic and Launder[8] as follows:

$$\overline{u_i u_j u_k} = -C_s \frac{k}{\epsilon}\left(\overline{u_i u_\ell}\frac{\partial \overline{u_j u_k}}{\partial x_\ell} + \overline{u_j u_\ell}\frac{\partial \overline{u_k u_i}}{\partial x_\ell} + \overline{u_\ell u_k}\frac{\partial \overline{u_i u_j}}{\partial x_\ell}\right)$$
$$C_s = 0.11 \tag{1}$$

The pressure-strain correlation term π_{ij} can be decomposed into a slow term $\pi_{ij,1}$, a rapid term $\pi_{ij,2}$, and a near-wall term $\pi_{ij,w}$. Incorporating the nonlinear effect, Sarkar and Speziale[9] developed a quadratic nonlinear model for the slow pressure-strain correlation term as follows:

$$\pi_{ij,1} = -\epsilon\{C_1' b_{ij} - C_2' [b_{ik}b_{kj} - (II_b/3)\delta_{ij}]\}$$
$$II_b = b_{ik}b_{ki}, \qquad C_1' = 3.4, \qquad C_2' = 4.2 \tag{2}$$

where b_{ij} is the anisotropy tensor defined by $b_{ij} = 0.5\overline{u_i u_j}/k - \delta_{ij}/3$.

The rapid term is represented by the model of Launder et al.[10] The near-wall term $\pi_{ij,w}$ is further decomposed into $\pi_{ij,w1}$ and $\pi_{ij,w2}$, which are approximated by the models of Shir[11] and Gibson and Launder,[12] respectively.

Finally, the dissipation rate equation is taken as

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_\ell}\left(C_\epsilon \frac{k}{\epsilon}\overline{u_\ell u_m}\frac{\partial \epsilon}{\partial x_m}\right) + \frac{\epsilon}{k}\left(C_{\epsilon 1}P - C_{\epsilon 2}\,\epsilon\right)$$
$$C_\epsilon = 0.15, \qquad C_{\epsilon 1} = 1.44, \qquad C_{\epsilon 2} = 1.92 \tag{3}$$

In the present study, adopting the same corrections as in Park and Chung,[3] the diffusive coefficients C_s in Eq. (1) and C_ϵ in Eq. (3) are replaced by modified coefficients C_s' and C_ϵ':

$$C_s' = C_s\frac{1}{1 + aH(N^2)\tau_v/\tau_c}, \qquad C_\epsilon' = C_\epsilon\frac{1}{1 + aH(N^2)\tau_v/\tau_c} \tag{4}$$

Here, $H(N^2)$ is the Heaviside step function ($H = 1$ when $N^2 \geq 0$, and $H = 0$ when $N^2 < 0$).

In addition, the isotropic decay rate constant $C_{\epsilon 2}$ in Eq. (3) is replaced by

$$C_{\epsilon 2}' = C_{\epsilon 2}\frac{1}{1 + b\tau_v/\tau_c} \tag{5}$$

Here, the model constants a and b were proposed to be 0.12 and 0.5, respectively, by Park and Chung.[2] In the present study, however, it was found that $b = 0.15$ gives better predictions. Note that theoretically $C_{\epsilon 2}'$ is a bounded value in a range of $1.4 < C_{\epsilon 2}' < 2.0$.[13]

Computations and Discussion of the Results

The governing equations are solved using a variant of the line-by-line SIMPLE procedure, in which the velocity components are stored midway between the pressure storage locations. All of the Reynolds stresses are evaluated at the scalar node points. The hybrid differencing scheme is used with 75 × 78 fine grids to reduce false diffusion. At the inlet plane, the streamwise mean velocity profile was given by the experimental data. At the outlet, gradients of flow properties in the flow direction are zero, i.e., $d\phi/dx = 0$, where ϕ is the flow property in question. This outlet is located at 60 times the step height downstream from the backward-facing step. At the wall boundaries an improved wall treatment proposed by Ciofalo and Collins[14] was used to calculate local sublayer thickness y_v^+ and friction velocity u_τ^2.

Present numerical computations and experimental data of Pronchick[6] are compared in Figs. 1–3. Figure 1 shows the velocity profiles at several downstream locations. It can be seen that the curvature RSM gives better results, especially at locations of $X/H = 4$ and 6 in the recirculating region. The predicted reattachment length by the conventional RSM is $5.2H$. The curvature RSM yields the reattachment length to be $6.4H$, which is much closer to the experimental result of $6.78H$. Figures 2 and 3 show comparisons between predictions and experimental profiles of the Reynolds shear stress $\overline{uv}$ and the streamwise normal stress $\overline{u^2}$. The predictions of $\overline{uv}$ and $\overline{u^2}$ are greatly improved by the curvature RSM. According to an experiment[15], the Reynolds shear stress and normal stresses are reduced by the streamwise curvature in a convex curved region. The success of the present curvature model is attributed to its correct adaptation to the variation of streamline curvature throughout the recirculating flowfield. Next, the

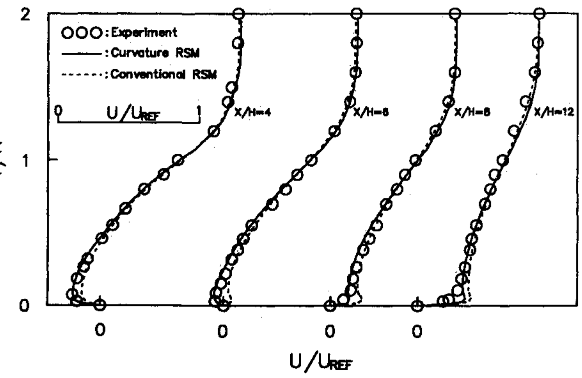

Fig. 1 Comparisons of predicted mean velocity profiles with experiment.[6]

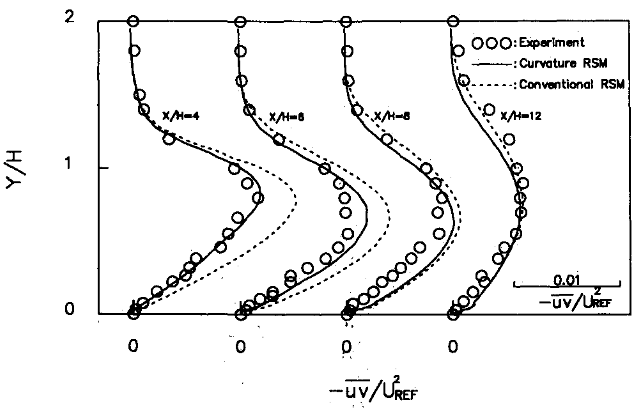

Fig. 2 Predicted Reynolds shear stress profiles compared to experiment.[6]

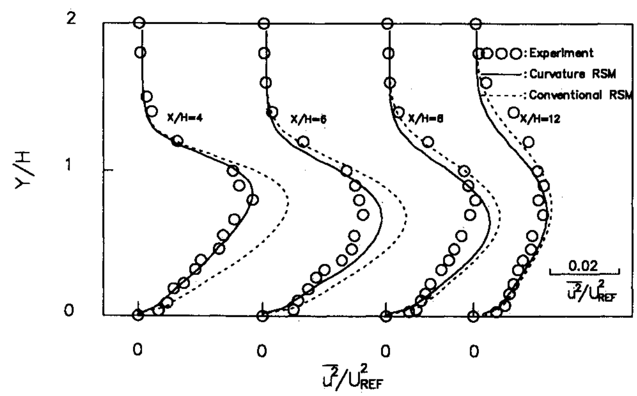

Fig. 3 Streamwise turbulent fluctuation profiles $\overline{u^2}$—predictions and experiment.[6]

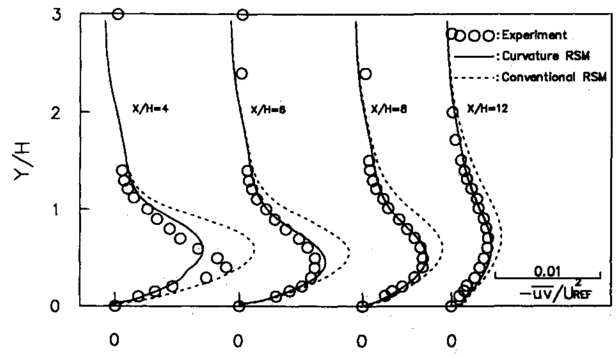

Fig. 4 Predicted Reynolds shear stress profiles compared to experiment.[7]

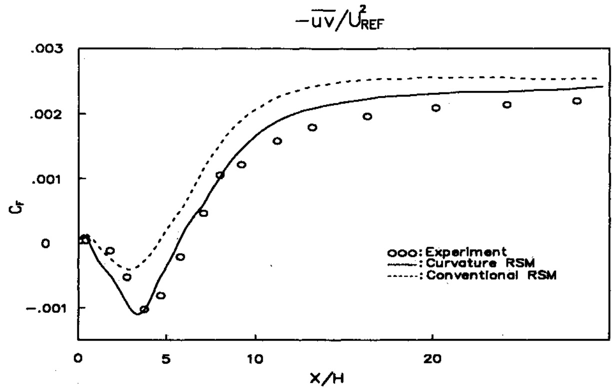

Fig. 5 Variation of skin-friction coefficient along the downstream distance—predictions and experiment.[7]

experiment of Driver and Seegmiller[7] has been computed with the two Reynolds stress models. Although the comparison is not shown for this case, the agreement between the predicted mean velocity profiles and the experiment, is very similar to that in Fig. 1. The computed reattachment length is about $5.6H$ in comparison with the measured value of $6.0H$, but the conventional RSM yields a much shorter length of about $4.5H$. Figure 4 compares the computed and measured profiles of the Reynolds shear stress. Again, very satisfactory predictions are obtained with the present curvature RSM. Finally, Fig. 5 depicts the variation of the skin friction along the downstream distance, which shows a remarkable result of the curvature RSM. Particularly, the skin friction in the separated flow region is far better estimated by the curvature model.

Conclusions

Two different sets of velocity field measurements in the recirculating flow around a backward-facing step in which the streamlines are highly curved are used to resolve the dispute about the necessity of streamline curvature correction to the second-order Reynolds stress model. The basic form of the Reynolds stress model was formulated with a nonlinear return-to-isotropy model of Sarkar and Speziale instead of the Rotta model.[16] Then curvature corrections were made to third-order transport terms in the equations of $\overline{u_i u_j}$ and ϵ and to the decay rate constant in the ϵ equation. The results show that such curvature corrections improve the predictions of Reynolds shear stress and streamwise normal stress. The improvement in the computations of $\overline{uv}$, in particular, was significant in the recirculating region. Consequently, the mean velocity profiles, the reattachment length, and the skin friction are better predicted by the curvature Reynolds stress model.

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New Wall-Reflection Model Applied to the Turbulent Impinging Jet

T. J. Craft*
Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique, Toulouse, France
and
B. E. Launder†
UMIST, Manchester M60 1QD, England, United Kingdom

Introduction

WITH the continuing rapid decrease in the cost of computer resources, computations of turbulent flows are progressively raising the level of physical models employed to represent turbulent momentum transport. Although, at present, most computations of aerodynamic flows still use models based on an effective turbulent viscosity, a growing minority adopt schemes that, instead, solve a set of rate equations for the turbulent stresses and, where appropriate, for the turbulent heat fluxes. Models of this type are known as second-moment (or second-order) closures. So far they have been applied mainly to free flows or to flows broadly parallel to walls and have established a track record of out performing eddy viscosity models, particularly where the streamlines are curved.

In second-moment closures for the turbulent stress field, a wall-reflection correction is conventionally added to the model of the pressure-strain correlation φ_{ij} in computing flow near walls. Its role is to reduce the level of turbulent velocity fluctuations normal to the wall and, through the strong intercoupling among the Reynolds stress components, to reduce generally the level of turbulent mixing. The various models of this process have been designed to produce approximately the correct relative levels of the Reynolds stresses in the near-wall, local-equilibrium region of the turbulent boundary layer or some other similar shear flow directed *parallel* to the wall (see Shih and Lumley[1] and Gibson and Launder[2]). When, however, the scheme of Ref. 2 was applied to the axisymmetric impinging jet[3,4] (see the broken lines in Figs. 1 and 2), it led to excessive levels of the turbulent stresses in the vicinity of the stagnation point. This anomalous behavior of the wall correction near stagnation points has also been recently noted by Murakami et al.[5] in a study of a three-dimensional buoyant jet in an enclosure and by Lea[6] in an in-cylinder flow. The present contribution proposes an alternative formulation of the part of the model giving rise to the aforementioned aberrant behavior.

Analysis

The pressure-strain correlation $\overline{p(\partial u_i/\partial x_j + \partial u_j/\partial x_i)}/\rho \equiv \varphi_{ij}$ that, as its name suggests, is the time-averaged product of the turbulent kinematic pressure and strain rate plays a crucial role in the budget of the Reynolds stress tensor $\overline{u_i u_j}$. Since, in an incompressible flow, its trace is zero, it serves to redistribute energy among the normal stresses and to diminish the correlation between off-diagonal components. There are two

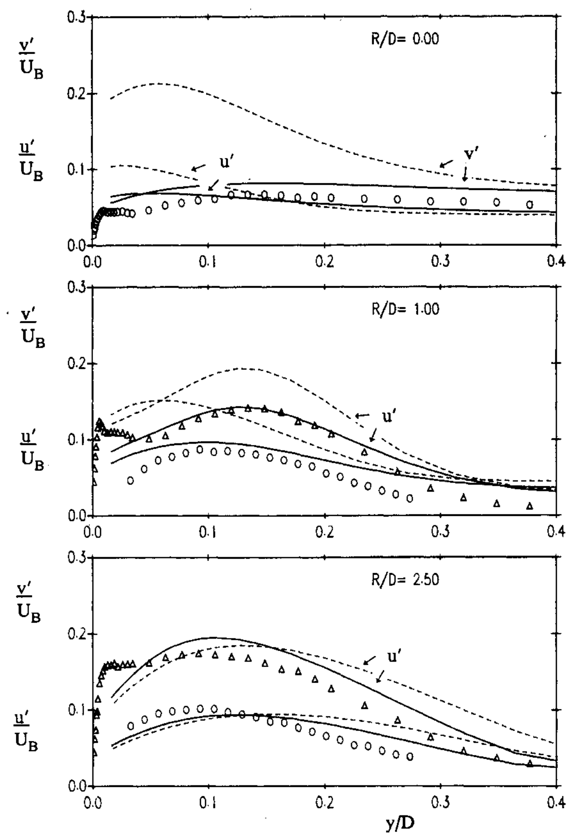

Fig. 1 Profiles of rms velocity fluctuations normal to plate (v') and radially (u') normalized by bulk velocity in pipe U_B: Δ, ○ u', v' experiment[11]; lines, computations: basic model, Eq. (4), --------; and new wall reflection model, Eq. (5), ———.

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*Currently, Research Associate, Mechanical Engineering Department, UMIST, Manchester M60 1QD, England, UK.

†Professor, Mechanical Engineering Department, P.O. Box 88.